Numbers, Operations and Relationships

Exponents

First law: Multiplication

$$x^m \times x^n = x^{m+n}$$

When powers with the **same basis multiply** → we **add** the exponents

Examples:

a) $3^4 \times 3^3 = 3^{4+3} = 3^7$
b) $x^4 \times x^3 = x^{4+3} = x^7$
c) $a^2 \times a^3 = a^{2+3} = a^5$
d) $x.x^3 = x^1 \times x^3 = x^4$
e) $(p^2)(p^4) = p^2 \times p^4 = p^6$

TAKE NOTE:
This does not count for powers with different bases!

Let's see why…

- $2^3 \times 3^2 = 8 \times 9 = 72$ **NOT** $2^3 \times 3^2 \neq 6^5$ $(6^5 = 7776)$
- $2^3 \times 2^2 = 2^5 = 32$ **NOT** $2^3 \times 2^2 \neq 4^5$ $(4^5 = 1\ 024)$

Second law: Division

$$x^m \div x^n = x^{m-n} \quad \textbf{OR} \quad \frac{x^m}{x^n} = x^{m-n}$$

When powers with the **same basis divide** → we **subtract** the exponents

Examples:

a) $3^6 \div 3^2 = 3^{6-2} = 3^4$

b) $\frac{x^4}{x} = x^{4-1} = x^3$

c) $a^{10} \div a^3 = a^{10-7} = a^7$

d) $\frac{2^5}{2^2} = 2^3$

e) $\frac{6a^3b^6}{3ab^4} = 2a^2b^2$

f) $\frac{6x^5y}{4x} = \frac{3x^4y}{2}$

g) $\frac{2x^{10}y^3z}{4xy^2z} = \frac{x^9y}{2}$

h) $\frac{10p^2q^2}{10p^2q^2} = 10^0p^0q^0 = 1$

i) $\frac{9a^6b^8}{27a^{12}b^6} = \frac{1b^2}{9a^6}$

Anything to the power of zero is always = 1

Third law: To raise a power to a power

$$(x^m)^n$$

- When you raise a power (x^m) to a greater power $(x^m)^n$, the exponents multiply with each other
- This is basically distributive law for exponents!

Examples:

a) $(x^5)^2 = x^{5\times 2} = x^{10}$ because $(x^5)^2 = x^5 \times x^5 = x^{5+5} = x^{10}$

b) $(2a^2)^3 = 2^{1\times 3}\mathrm{a}^{2\times 3} = 2^3\mathrm{a}^6 = 8\mathrm{a}^6$ because $(2a^2)^3 = 2a^2 \times 2a^2 \times 2a^2$
$= 2^{1+1+1}\mathrm{a}^{2+2+2}$
$= 2^3\mathrm{a}^6$
$= 8\mathrm{a}^6$

c) $(a^3)^5 = a^{15}$

d) $(2a^3b^4)^3 = 2^3a^9b^{12} = 8a^9b^{12}$

e) $(3ab^2)^2 = 9a^2b^4$

f) $(a^4b^7)^4 = a^{16}b^{28}$

g) $a^2(a^3)^2 = a^2 \times a^6 = a^8$

h) $2a^3(3a^2)^2 = 2a^3 \times 9a^4 = 18a^7$

Fourth law: Everything to the power of zero is equal to one

$$x^0 = 1$$

Let's look at why…
Look what happens when you divide two identical numbers with each other:

$$5 \div 5 = 1$$

No let's get a little bit more technical:

$$5 \div 5 = 5^1 \div 5^1 = 5^{1-1} = 5^0 = 1$$

∴ everything to the power of zero is always one!

Examples:

a) $(abc)^0 = 1$

b) $a^2b^4c^0 = a^2b^4$

c) $2(ab)^0 = 2\times 1 = 2$

Fifth law: To determine the power of a surd (root)

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

Take the exponent on the "inside" and divide it with the power on the "outside"

Here is proof to why it works:

$$\sqrt{64} = 8 \textbf{ or } \sqrt{64} = \sqrt{2^6} = 2^{6\div 2} = 2^3 = 8$$
$$\sqrt{16} = 4 \textbf{ or } \sqrt{16} = \sqrt{2^4} = 2^{4\div 2} = 2^2 = 4$$
$$\sqrt{4} = 2 \textbf{ or } \sqrt{4} = \sqrt{2^2} = 2^{2\div 2} = 2^1 = 2$$

Examples:

a) $\sqrt{144a^{12}b^6} = 12a^6b^3$

b) $\sqrt[3]{27a^6b^{15}c^{21}} = 3a^2b^5c^7$

c) $\sqrt{\frac{81a^8b^{10}}{64a^6b^{20}}} = \frac{9a^4b^5}{8a^3b^{10}} = \frac{9a}{8b^5}$

Exercise:

Simplify the following expressions:

a) $3x^2 \times 2x^5$ (2)

k) $(x^2)^3$ (1)

b) $x^2y^2 \times x^3y^3$ (2)

l) $(3ab)^3$ (2)

c) $(2a^3b)(4ab^5)$ (3)

d) $3x^2 \times 2x^5y + y$ (4)

e) $\frac{y^6}{y^2}$ (1)

f) $\frac{6x^5}{3x}$ (2)

g) $\frac{2x^5y^3}{8xy^3}$ (3)

m) $2(2a^3)^2$ (2)

n) $-(4^2)^2$ (1)

o) $[2(3xy)]^2$ (4)

p) $\left(\frac{2m}{3p}\right)^2$ (2)

q) $\frac{(a^2)^3a^2a^3}{a^{10}}$ (3)

h) $\frac{5xy^2}{5xy^2}$ (1)

r) $\frac{(x^2)^3 \div (x^3)^2}{x^0}$ (4)

i) $\sqrt{25x^6 \times 2^2 x^8 y^{10}}$ (3)

s) $\frac{x^4 y \times x^2 y}{x^3 y^2}$ (3)

j) $\sqrt[4]{625x^{12}y^{28}}$ (3)

t) $5xy^0 \times (7x)^0 \times 2x(y)^0$ (3)

MEMO

Please take note: *This memo contains a lot of detail so that it is clear where the answer comes from, pupils are not required to show as many steps, therefore, refer to the mark allocation as to which steps are necessary.*

a) $3x^2 \times 2x^5$
$= (3\times 2)\times(x^2 \times x^5)$
$= 6\times x^{2+5}$
$= 6x^7$

✓ 6 ✓ x^7

b) $x^2y^2 \times x^3y^3$
$= (x^2 \times x^3)\times(y^2 \times y^3)$
$= x^{2+3}\times y^{3+3}$
$= x^5y^6$

✓ x^5 ✓ y^6

c) $(2a^3b)(4ab^5)$
$= 2a^3b\times 4ab^5$
$= (2\times 4)\times(a^3\times a^1)\times(b^1\times b^5)$
$= 8\times a^{3+1}\times b^{1+5}$
$= 8a^4b^6$

✓ 8 ✓ a^4 ✓ b^6

d) $3x^2\times 2x^5y + y$
$= (3x^2\times 2x^5y) + y$
$= (6\times x^{2+5}\times y) + y$
$= 6x^7y + y$

BODMAS
There is no exponent laws for addition and subtraction
✓ 6 ✓ x^7 ✓ y ✓ $+y$

e) $\frac{y^6}{y^2}$
$= y^{6-2}$
$= y^4$

✓ Answer

f) $\frac{6x^5}{3x}$
$= (6 \div 3)\times(x^5 \div x^1)$
$= 2\times x^{5-1}$
$= 2x^4$

✓ 2 ✓ x^4

g) $\frac{2x^5y^3}{8xy^3}$
$= (2\div 8)\times(x^5\div x^1)\times(y^3\div y^3)$
$= \frac{1}{4}\times x^{5-1}\times y^{3-3}$
$= \frac{x^4}{4}$

Be careful, $2 \div 8 = \frac{1}{4}$ and not 4
✓ $\frac{1}{4}$ ✓ x^4 (in the numerator) ✓ $y^0 = 1$

h) $\frac{5xy^2}{5xy^2}$
$= 1$

Two identical number dividing with each other is always equal to 1
✓ Answer

i) $\sqrt{25x^6 \times 2^2 x^8 y^{10}}$
$= \sqrt{100x^{14}y^{10}}$ — First simplify underneath the root
$= 10 \times x^{14\div 2} \times y^{10\div 2}$ — Apply the fifth law
$= 10x^7y^5$ — ✓ 10 ✓ x^7 ✓ y^5

j) $\sqrt[4]{625x^{12}y^{28}}$
$= 5 \times x^{12\div 4} \times y^{28\div 4}$ — Apply the fifth law
$= 5x^3y^8$ — ✓ 5 ✓ x^3 ✓ y^8

k) $(x^2)^3$
$= x^{2\times 3}$ — Apply the fourth law
$= x^6$ — ✓ Answer

l) $(3ab)^3$
$= 3^{1\times 3} \times a^{1\times 3} \times b^{1\times 3}$ — Apply the fourth law
$= 27a^3b^3$ — ✓ 27 ✓ a^3b^3

m) $2(2a^3)^2$
$= 2 \times 2^{1\times 2} \times a^{3\times 2}$ — Apply the fourth law
$= 8a^6$ — ✓ 8 ✓ a^6

n) $-(4^2)^2$
$= -1 \times 4^{2\times 2}$
$= -4^4$
-256 — ✓ Answer

o) $[2(3xy)]^2$
$= 2^{1\times 2} \times (3xy)^{1\times 2}$ — ✓ 2^2 ✓ $(3xy)^2$
$= 4 \times 3^{1\times 2}x^{1\times 2}y^{1\times 2}$ — Work from the outside towards the inside to remove the brackets
$= 36x^2y^2$ — ✓ 36 ✓ x^2y^2

p) $\left(\frac{2m}{3p}\right)^2$
$= \frac{2^{1\times 2}m^{1\times 2}}{3^{1\times 2}p^{1\times 2}}$ — Apply the fourth law
$= \frac{4m^2}{9p^2}$ — ✓ Numerator ✓ Denominator

q) $\frac{(a^2)^3a^2a^3}{a^{10}}$
$= \frac{a^{2\times 3}a^2a^3}{a^{10}}$ — First remove the bracket by applying the fourth law
✓ a^6

$= \frac{a^{6+2+3}}{a^{10}}$ — Simplify the numerator

$= \frac{a^{11}}{a^{10}}$ — ✓Numerator

$= a$ — ✓Answer

r) $\frac{(x^2)^3 \div (x^3)^2}{x^0}$

$= \frac{x^{2\times3} \div x^{3\times2}}{1}$ — ✓$y = 1$ (but they do not need to show it)

$= x^6 \times x^6$ — ✓✓x^6

$= x^{6+6}$

$= x^{12}$ — ✓Answer

s) $\frac{x^4 y \times x^2 y}{x^3 y^2}$

$= \frac{x^{4+2} \times y^{1+1}}{x^3 y^2}$ — First simplify the numerator

$= \frac{x^6 \times y^2}{x^3 y^2}$ — ✓x^6 ✓y^2

$= x^{6-3} \times 1$ — $y^{2-2} = y^0 = 1$

$= x^3$ — ✓ Answer

t) $5xy^0 \times (7x)^0 \times 2x(y)^0$

$= 5x(1) \times 1 \times 2x(1)$

$= 5x \times 1 \times 2x$

$= 10x^2$